Applying Piecewise Linear Characteristic Curves in District Energy Optimisation

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Abstract:

Representing nonlinear curves as piecewise elements allows complex systems to be optimised by linear programs. Piecewise linearisation has been recently introduced in the context of distributed energy system optimisation. It is an efficient technique for representing non-linear technology behaviours in linear optimisation models, which are favourable in district energy optimisation models, owing to their speed and ability to handle large numbers of design variables. This paper describes a method of automating the creation of piecewise elements of technology performance curves for minimum fit error. The results show an objective function value improvement at a relatively large penalty in solution time: from 1.6 times to 58 times longer than describing technologies as having a single value for efficiency (SVE). We show that within the context of common technology performance curves, three breakpoints yield sufficiently accurate results and any returns are diminishing beyond that. Even at three breakpoints, it is evident that the placement of breakpoints along a curve significantly influences solution time, in a way for which it is not possible to account in automation. But, large savings can be made by automation by including a constraint to ensure piecewise curves have a strictly increasing/decreasing gradient. This avoids the use of special ordered sets, simplifying model generation and the number of non-continuous variables. SVE models provide a less realistic solution and application of nonlinear consumption curves ex-post shows them to be ultimately more expensive systems than their piecewise counterparts. However, this ex-post analysis applied to SVE models is a good compromise for feasibility level analyses, where whole system cost is key. However, investment decisions and operation schedules are markedly affected by consumption curve representation. Thus, the use of piecewise linearisation is beneficial for detailed design, particularly if automation of breakpoint allocation can help solve the issue of model convergence.

Keywords:

District energy, Mixed Integer Linear Programming, Optimisation, Piecewise Linearisation.

1. Introduction

The application of energy system optimisation at a district level can lead to tangible infrastructural decisions by designers. Considering this, the energy system must be realistically represented, such that results can reliably direct the decision-making process. Most current models use mixed-integer linear programming (MILP) to optimise energy systems at a district-scale. In addition to allowing fast solutions of large-scale problems, MILP models can efficiently represent energy distribution networks. However, they are unable to handle non-linear characteristics of energy supply technologies. Cooling technologies particularly exhibit nonlinearity in their operation, when operating below nominal load and at different external/internal temperatures. Indeed, commercial properties were not considered for optimisation in MILP by [1] due to the need to model cooling technologies. Metaheuristic techniques are used to include system non-linearities, but can become intractable for large-scale problems, taking far longer than MILP to reach a reasonable solution [2].

By describing a non-linear curve of an energy supply technology as multiple, connected linear pieces, it is possible to compromise between model fidelity and computational efficiency. [3] showed that bicubic and cubic technology part-load curves could be represented in piecewise form. In fact, piecewise curves could contain up to ten pieces without significant effect on computational time. An important factor found in this study was the location at which pieces meet (the `breakpoints'), but this was not explored in detail. When comparing MILP and metaheuristics for operation schedule optimisation, [2] undertook piecewise linearisation. Six breakpoints were applied to linearise part load curves, specifically located at discontinuities and the point of maximum efficiency. The subsequent piecewise MILP model led to an objective function value similar to the same system optimised metaheuristically with non-linear curves. However, the choice of breakpoint positions was not tested against other configurations.

This paper extends both these studies, by considering a more complex case while investigating breakpoint positioning. A sequential least squares programming (SLSQP) algorithm is used to minimise the error between piecewise and nonlinear technology part-load consumption curves. This nonlinear optimisation is compared to both placing pieces equidistantly along the x-axis and to a single value for efficiency (SVE). At full load, all curves converge on the nominal efficiency of a technology, but at any part-load value it is possible to quantify the error between the "actual" nonlinear case and "expected" linearised cases. The minimisation of this error is compared to computational time penalty when applied to a district energy system case study.

2. Case study

A district planning case is considered, due to the non-negligible requirement for electricity, heating, and cooling when combining different building types. This district is notional and consists of 10 domestic properties, one large hotel, one large office, and one power plant (figure 1). Within the district, a range of technologies is available to meet demand of each energy type (table 1). Distribution networks exist for low voltage electricity, gas, and heat. Table 2 gives further information on attributes of each property type.

2.1. Chosen technologies

Multiple technologies exist to meet each type of energy demand. In this case study, the technology choice facilitates the need for optimisation, due to different energy interdependencies. Grid electricity (GE) and the boiler (NB), air source heat pump (AHP), and electric chiller (EC) can provide their respective energy demands without interdependency, but have relatively high generation costs. Solar photovoltaic (PV) and solar thermal (ST) panels benefit from government subsidies, such as the feed-in tariff, but have fixed output once maximum capacity has been selected and the available roof space is limited. CHP produces both heat and power simultaneously, giving a low generation cost but a high initial capital investment (including a district heat network), while the heat recovery absorption refrigerator (HRAR) can be powered by either waste heat or gas. Finally, storage facilities exist for each energy type. By decoupling supply and demand temporally, storage reduces the effect of interdependencies. Electricity can be produced by the CHP without worrying about heat demand, and the PV and ST supply can be maximised in the knowledge that all production can be effectively used on-site.

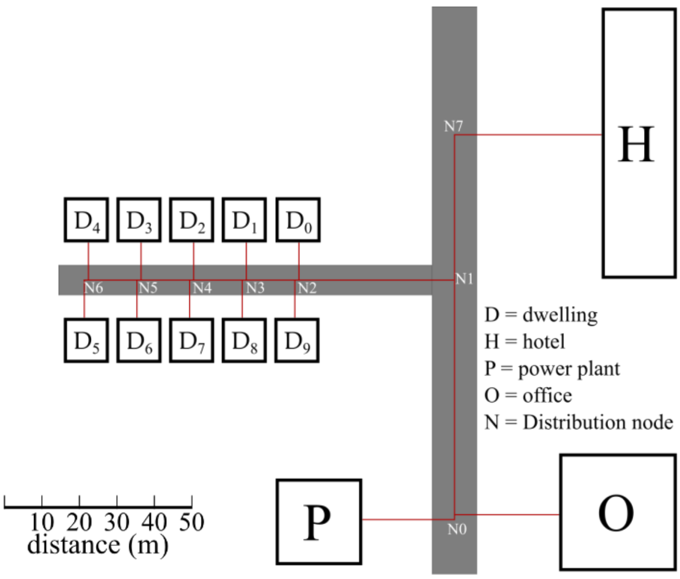


Figure 1. Graphical representation of case study district network.

Table 1. Model supply technologies and their consumption/production energy. E = electricity, G = gas, S = solar radiation, C = cooling, H = heating.

Technology	*AHP*	*EC*	*HRAR*	*CHP*	*NB*	*PV*	*ST*	*B*	*TES*
Consumption	E	E	G, H	G	G	S	S	E	H/C
Production	C	C	C	E, H	H	E	H	E	H/C

Table 2. Case study building characteristics.

		Dwelling	*Hotel*	*Office*	*Plant*
Annual energy demand (MWh)	*Electricity*	7.2	1595.5	481.3	0
	Heat	17.5	1641.6	86.5	0
	Cooling	0.0	1757.9	99.1	0
Available roof area (m^2)		130	1300	900	0
Available technologies		NB, PV, ST, B, TES	mCHP, NB, PV, HRAR, AHP, EC, ST, B, TES	NB, PV, ST, HRAR, AHP, EC, B, TES	CHP, GE

2.2. Data

To create a notional district, data on energy demand, technology characteristic curves, and costs have been brought together from multiple sources:

- The district is located in the South-East of England, UK. However, due to availability, U.S. Department of Energy representative building demand data [4] is used to acquire hourly heat, cooling, and electricity demand of representative buildings. Seattle, Washington climate conditions were chosen for climate similarity with London, UK.
- Characteristic curves for technologies are based on recommendations from Society of Heating, Air-conditioning and Sanitary Engineers of Japan (SHASE) [5]. It is assumed that energy supply technologies do not vary drastically between countries.
- Costs curves are calculated based on values given in the SPON'S mechanical and electrical services price book [6]. Storage device costs have been aggregated from online suppliers.

3. Methodology

3.1. Piecewise linearisation

Typically, in energy modelling, the efficiency of a technology is given as a single value, based on nominal conditions, e.g. [7–9]. In reality, efficiency varies depending on the output of the technology as a function of its maximum capacity, among other factors [10]. This nonlinearity can be addressed by metaheuristic optimisation, which allows nonlinear inputs. However, given the non-deterministic nature of metaheuristic methods, mathematical programming, usually in the form of MILP, is still the dominant energy modelling method. To integrate nonlinear technology characteristics with MILP, it is possible to approximate a nonlinear curve by segmenting it into several straight lines. These straight lines create a linear, but discontinuous curve which can be handled in a linear program. For application within the MILP environment, two approaches will be discussed in this section: special ordered sets and constraint bound.

3.1.1. Special ordered sets

Special ordered sets of type 2 (SOS2), first introduced by [11], are often used in MILP to piecewise linearise. Sampling points ('breakpoints') x_i $(i = 1, \ldots, n)$ are defined along a curve, including the start and end of the curve, with corresponding y-axis values $f(x_i)$ $(i = 1, \ldots, n)$ (figure 2). A continuous decision variable, α_i is associated with each breakpoint i, such that $\alpha_i \in [0, 1]$ $(i = 1, \ldots, n)$. By defining the α variables to be SOS2, constraints are applied so only two adjacent α variables can be non-zero at any one time. For any decision variable x, the corresponding decision variable value $f(x)$ is calculated by interpolating from adjacent breakpoints $(x_i,\ f(x_i))$ and $(x_{i+1},\ f(x_{i+1}))$, based on the relative weighting applied by α_i and α_{i+1}. In energy planning, part-load efficiency is a function of two decision variables: load rate and maximum capacity. If maximum capacity is a *discontinuous* variable, then an SOS2 can be described for discrete values of capacity.

If maximum capacity is a *continuous* variable, more complex methods are required, but special ordered sets are still applicable. The 3D surface describing the relationship between maximum capacity (x), load-rate (y) and consumption $(f(x,y))$ can be discretised. The most common approach is to have n breakpoints $x_1, \ldots, x_n$ on the x axis and m sampling points $y_1, \ldots, y_m$ on the y axis [12]. $f(x,y)$ is evaluated for each breakpoint. Any point $(\bar{x},\bar{y})$ can be evaluated within the rectangle bounded by (x_i,y_j), (x_{i+1},y_j), (x_i,y_{j+1}), and (x_{i+1},y_{j+1}), which contains two triangles created by its diagonal $[(x_i,y_j),\ (x_{i+1},y_{j+1})]$ (figure 3). By convex combination of the function values evaluated at the vertices of the triangle containing $(\bar{x},\bar{y})$, $f(\bar{x},\bar{y})$ can be ascertained.

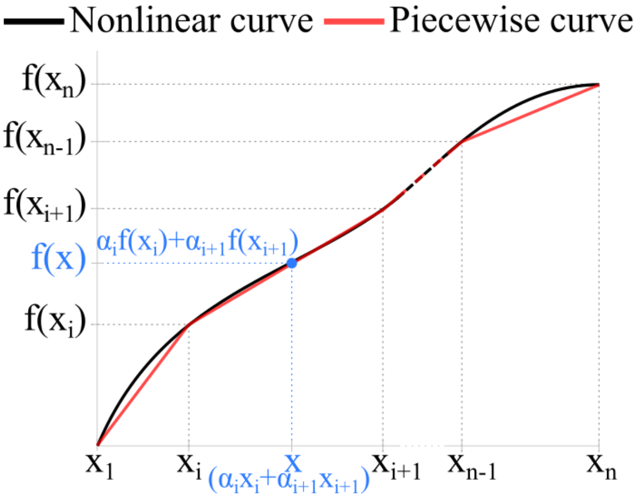


Figure 2. Graphical representation of SOS2 piecewise linearisation. $f(x)$ is the sum of weighted values $\alpha_i f(x_i)$ and $\alpha_{i+1} f(x_{i+1})$, with all other values of α being zero.

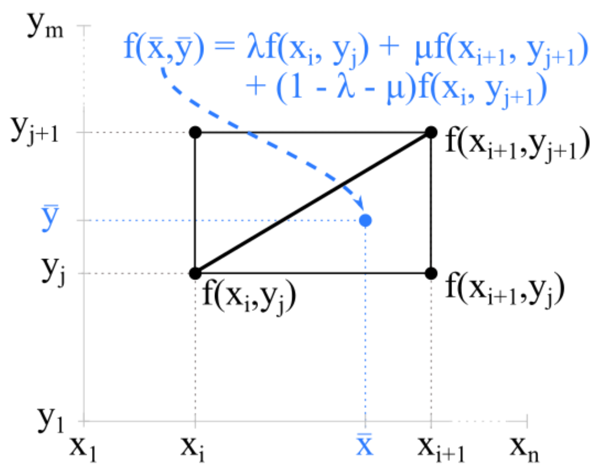


Figure 3. Graphical representation of 3D piecewise linearisation. $f(\bar{x},\bar{y})$ is the sum of weighted decision variables λ and μ applied to $f(x_i,y_j)$, $f(x_{i+1},y_{j+1})$ and $f(x_{i+1},y_j)$.

3.1.2. Bound by constraints

In creating special ordered sets, many new decision variables are defined, more so when a 3D surface exists. This will inevitably increase computational time, perhaps beyond what is feasible for the given problem. In certain cases, it is also possible to force a continuous decision variable to follow a piecewise curve, by applying constraints of the form $y = mx + C$, as depicted in figure 4a. The constraint lines intersect the nonlinear curve where the gradient, m, equals the curve instantaneous gradient. In the case of energy systems, the global minimum will only exist where each technology has chosen to minimise its consumption at every given value of energy output. This means that the consumption curve given in figure 4a will always follow the lower bound, which describes the piecewise curve. However, if the gradient of the technology characteristic curve is not strictly increasing/decreasing, this method cannot function. Figure 4b shows that certain lines describing the piecewise curve will override others at incorrect segments of load rate, due to the changing direction of gradient. Here, the consumption curve does not describe the piecewise curve. Although limited in its use cases, this method can also be extended easily to the 3D case, where the constraints are of the form $f(x,y) = my + Cx$, given a maximum capacity (x) and load-rate (y). On inspection of the characteristic curves used in this study, most met the gradient criterion for this method. The only technology which did not was the CHP, which has an undulating gradient when describing both its gas consumption and its heat output (figure 7). However, as will be discussed in the next section, it is possible to account for this when optimising the piecewise curves, to allow the bound by constraints method to viably be used for solving the given problem.

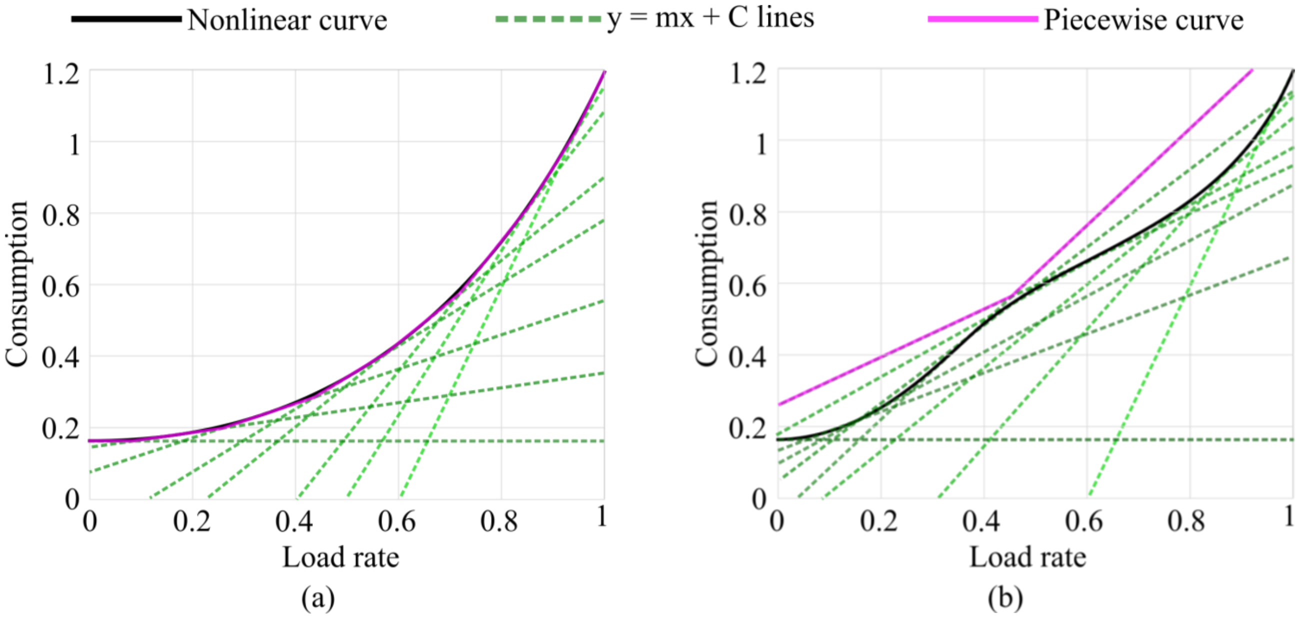


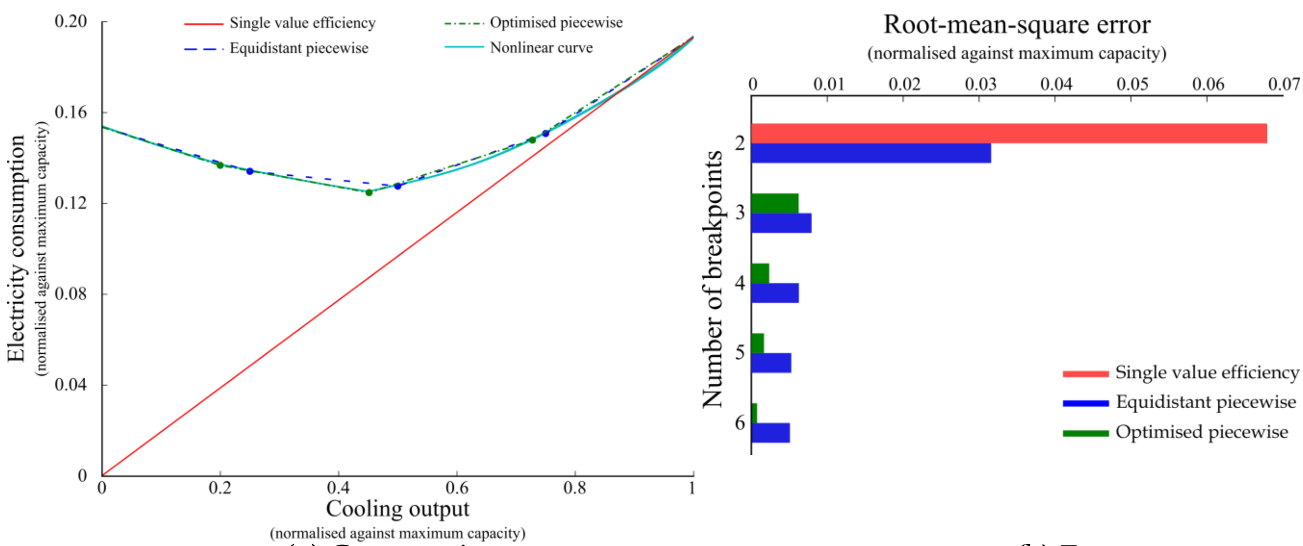
Figure 4. Application of bounding a technology nonlinear curve under multiple straight lines to create a piecewise linear curve. (a) shows its effective use on a curve of continuously decreasing gradient, while (b) shows its ineffectiveness when applied to a more complex curve.

3.2. Optimisation

For a limited number of breakpoints, there will be at least one optimal placement to describe a piecewise curve that best fits the nonlinear curve. The process of locating these breakpoints optimally can be simplified by automation. The piecewise curve with least error relative to the nonlinear curve, for a given number of breakpoints, can be ascertained when optimising. Additionally, the constraint that the gradient must be strictly increasing or decreasing can be applied, creating piecewise curves which meet the requirements set out in section 3.1.2.

Breakpoint allocation is undertaken during model pre-processing, by parameter optimisation. Previous studies [13], [14] have used heuristic algorithms to piecewise linearise. Here, we have used SLSQP [15] to minimise the root-mean-square error between each nonlinear curve and its piecewise counterparts. To improve the chances of reaching the global optimum, 20 runs were undertaken for each minimisation. This process took 17.1 seconds to optimise 108 piecewise curves describing characteristics of 8 technologies (27 nonlinear curves, three to six breakpoints). For the case of the EC, figure 5a shows the resulting 5-breakpoint curve. Curve fit is better when breakpoints are optimised, most notably in the trough. Any form of piecewise linearisation is an improvement on the SVE case, although there is continual improvement on error minimisation when optimising breakpoint location, as figure 5b depicts for the EC.

Some technologies, such as the boiler, have a relatively static efficiency over the operating range. In this case, there is little advantage to piecewise linearise, and even less reason to undertake parameter optimisation. Cooling technologies tend to function more nonlinearly. This nonlinearity can be a barrier to including cooling in a linear program [1], although it is usually considered to be caused by system temperatures rather than variable load-rate. It is evident from figure 5a that the EC acts nonlinearly with variable load rate. However, figure 6 shows that this nonlinearity is not as pronounced for other cooling supply technologies, unless operating at low load rates. Below a distinct discontinuity, the energy consumption becomes constant, irrespective of output. For the CHP, there is a reasonable disparity between the realistic operation and SVE, particularly when considering the heat to power ratio (figure 7). The CHP characteristic curves are also not strictly increasing/decreasing, the result of which can be seen in the difference between the two optimised curves, a difference that is not apparent for the other technologies. However, the difference is relatively small, becoming non-negligible only for parts of the heat to power ratio (HTP) curve.



(a) Consumption *(b) Error*

Figure 5. Comparison of different methods to describe electricity consumption of an EC, from nonlinear to SVE. (a) shows consumption curve and piecewise linearisation with five breakpoints, (b) shows root-mean-square error between each all methods and the nonlinear curve.

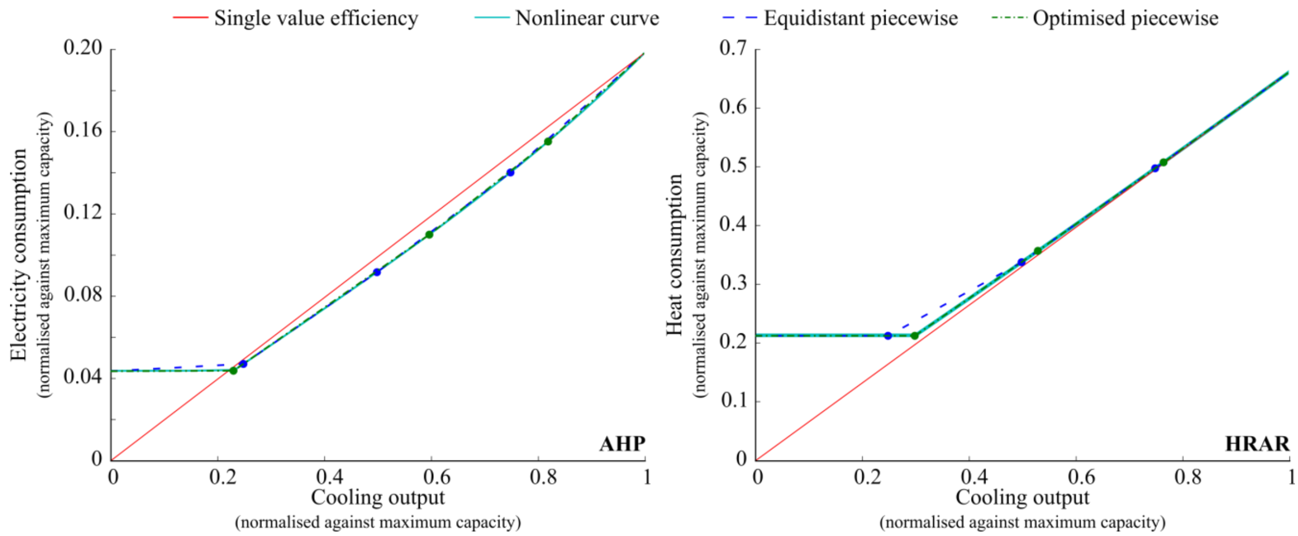


Figure 6. Comparison of different methods for describing the primary fuel consumption of an AHP and HRAR, from nonlinear to SVE, at different load rates. Piecewise curves have five breakpoints.

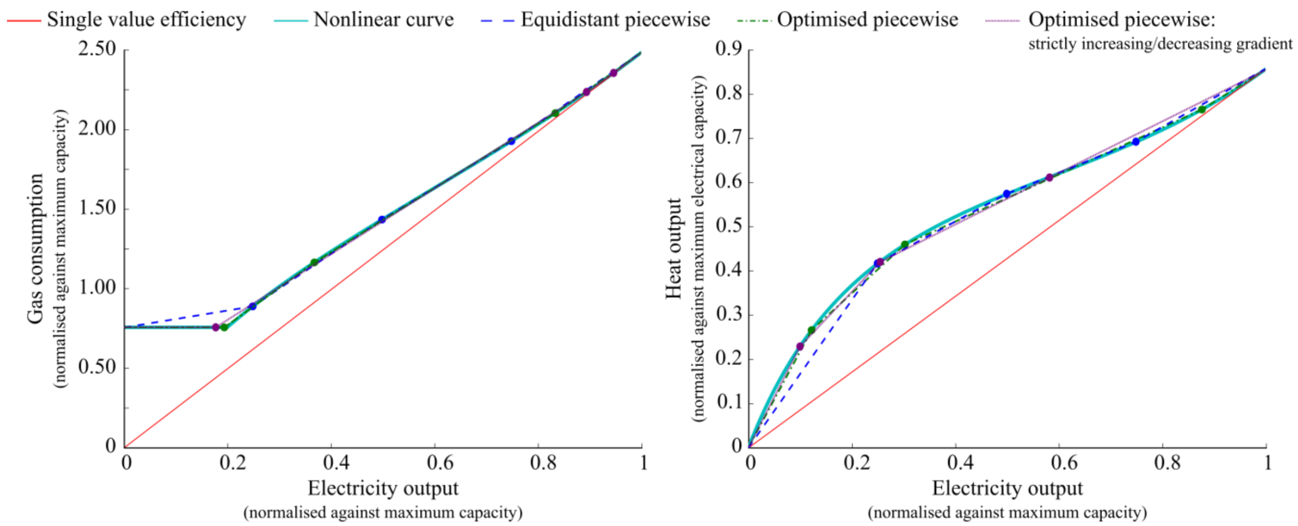


Figure 7. Comparison of different methods for describing the gas consumption and heat output of a CHP, from nonlinear to SVE, at different load rates. Five breakpoints given for piecewise curves.

As with cooling, the performance of thermal storage is primarily temperature dependant [16]. Varying load rates do also have an effect, due in part to the use of pumps during charge/discharge [17], but also due to thermal stratification required for minimal heat loss. If the flow rate of charge/discharge is too high, it will likely disrupt the stratified layers in the tank, leading to mixing and associated exergetic losses [18]. As temperature dependence is not considered in this study, nonlinear characteristics of storage technologies are not included. However, thermal energy flow is limited for the tanks, to simulate avoiding mixing effects.

3.3. Model configuration[1]

The case study was modelled in Calliope (https://www.callio.pe/), an open-source modelling framework which uses a python-based toolchain [19]. MILP optimisation was run via CPLEX [20], with a 3% mixed integer optimality gap tolerance on a 64-bit Windows 7 operating system with 2.50 GHz Intel Xeon E5-2680 v3 processor and 64GB RAM. Multiple model configurations were run, for different demand seasons, linearisation techniques, and breakpoints of piecewise linearisation (figure 8). The objective function throughout was minimisation of capital and operational costs, combined. Ex-post, error due to linearisation techniques was calculated.

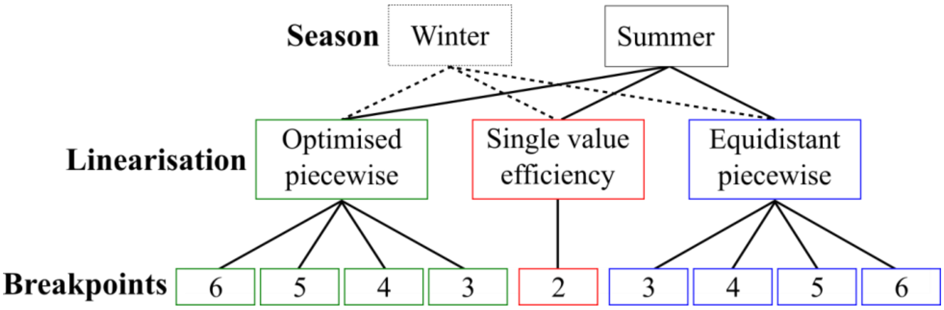


Figure 8. Configurations of modelling runs.

3.4. Case study simplification

The initial district was to be modelled over all hourly timesteps in a year. This created a problem of a size that could not be handled by the testing hardware. To maintain model tractability, individual weeks were considered instead. Two separate weeks were chosen based on maximum heat requirement (week 1) and maximum cooling requirement (week 28). The initial network in figure 1 was also aggregated to the network seen in figure 9, reducing decision variables from 8,649,607 to 410,905. In doing this, all dwellings were merged into a single domestic property and the hotel and office were merged into a commercial property. Total energy demand and available roof area remained constant. These simplifications were necessary to run the model multiple times, such that all the configurations given in figure 7 could be analysed in a timely fashion.

Initially, SOS2 was chosen as the method for representing the piecewise curves, described in section 3.1.1. But, model convergence was poor, particularly when within 10% of the relaxed LP solution. To ensure that all relevant technologies could be piecewise linearised, constraint bounds, introduced in section 3.1.2., were applied. This leads to a greater error in describing the CHP curve, particularly at a greater number of breakpoints. After four breakpoints, it is not possible to reduce HTP curve error further, leading to double the error between SOS2 and constraint bounds at six breakpoints (figure 10). However, both methods still provide a low error, lower than their equidistant counterparts. The technology characteristics considered for piecewise linearisation were the CHP HTP and gas consumption; EC and AHP electricity consumption; and HRAR heat consumption. Other characteristics available were the boiler gas consumption and the pumps associated with distributing thermal energy from supply to demand. These characteristics were ignored due to the linearity of the former and the small scale of the latter.

[1] Model configuration files and ex-post analysis can be found at: https://github.com/brynpickering/piecewise-calliope

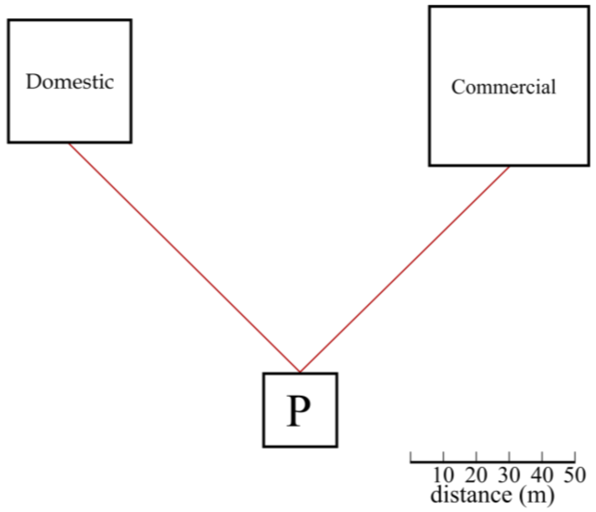


Figure 9. Graphical representation of case study district network, following simplification of district depicted in figure 1.

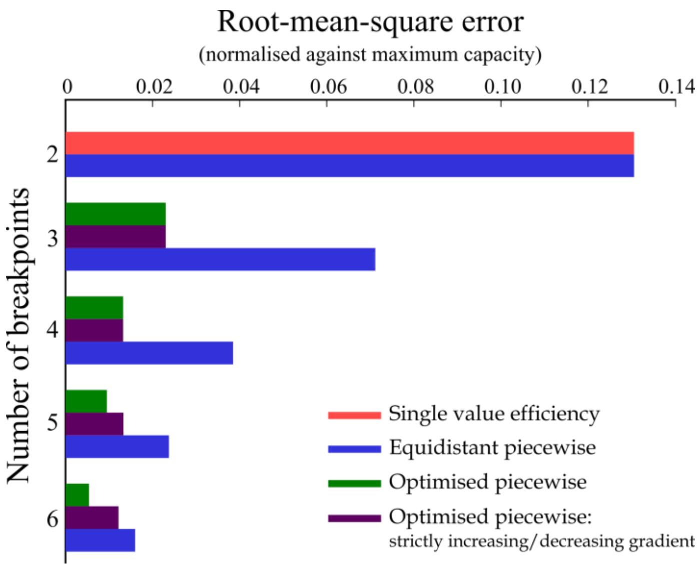


Figure 10. root-mean-square error between linearisation methods and the nonlinear characteristic curve of CHP HTP, for full range of breakpoints.

4. Results

4.1. System costs

Application of piecewise curves increases the objective function value by as much as 5.2%. Table 3 shows that differences in objective function value are small when increasing the number of piecewise breakpoints, with optimised curve averages of £4036 +1%/-0.5% in winter and -£2394 +0%/-0.6% in summer. The summer negative cost represents the ability for the system to gain more revenue from subsidies and export than it spends on investment and operation in that period. There are no equidistant solutions beyond three breakpoints due to model infeasibility. It is not possible to place constraints on breakpoint location when placing equidistantly. Thus, the strictly increasing/ decreasing gradient requirement for being bound by constraints cannot be met for CHP HTP and gas consumption.

Table 3. Objective function value in GBP for all run configurations. +NL = monetary cost incurred from applying nonlinear consumption curves ex-post, O = optimised, E = equidistant.

Breakpoints	*2*	*3*		*4*		*5*		*6*	
Linearisation	*SVE*	*O*	*E*	*O*	*E*	*O*	*E*	*O*	*E*
Winter									
Result	3989	4036	4048	4074	Fail	4019	Fail	4016	Fail
+NL	+465	+23	+12	+40	N/A	+32	N/A	+32	N/A
Summer									
Result	-2507	-2380	-2377	-2398	Fail	-2398	Fail	-2401	Fail
+NL	+294	-17	-28	-2	N/A	-1	N/A	0	N/A

Each linear model run has been compared to its nonlinear counterpart, by applying the relevant nonlinear consumption curves to the technology outputs obtained using the linear optimisation. In doing so, we can see the potential difference between "expected" (MILP objective function value) and "actual" (nonlinear consumption curves applied ex-post) system costs (+NL). Although the optimal SVE objective function value is lower than for piecewise models, the "actual" system costs end up being higher. +NL is 12% in both seasonal weeks for SVE, decreasing to less than 1% when including piecewise curves. In summer, this effect is most pronounced, where +NL reduces to zero at six breakpoints.

4.2. Model run time

While the accuracy of the objective function value is improved, piecewise linearised cases take much longer to solve than the SVE case (table 4). This is more the case in the summer week, which peaks at 17521 seconds (three breakpoints, equidistant), two orders of magnitude longer than the basic model. Even at the least number of breakpoints, the solution time is 2.5x and 14.9x longer than the basic model in winter and summer, respectively.

There is generally an increase in solution time with increased number of breakpoints, the only anomaly being the drastic decrease in model solution time between having five and six breakpoints in summer. Here, the model solves in less than half the time with an additional breakpoint. In this instance, the five-breakpoint case had solved within 10% of the relaxed LP 200 seconds sooner than the six-breakpoint case, but failed to converge on the last few percent for an extended period. Equidistant breakpoints decrease the solution time by a small amount in the winter week and increase it substantially in the summer week. As aforementioned, it is the final few percent of convergence that leads to the vastly inflated solution time.

Table 4. Model runtime in seconds for all configurations, including pre-processing and subsequent MILP solving in CPLEX. O = optimised, E = equidistant.

# of breakpoints	*2*	*3*		*4*		*5*		*6*	
Linearisation	*SVE*	*O*	*E*	*O*	*E*	*O*	*E*	*O*	*E*
Winter	366	926	610	880	Fail	847	Fail	1408	Fail
Summer	300	4483	17521	7202	Fail	15230	Fail	6816	Fail

4.3. Technology investment and operation

The change of objective function value when applying piecewise characteristic curves results from changes in both investment and operation. Varying the "penalty" for part load operation leads to different technology choices. For instance, in meeting cooling demand in the SVE case, the EC is chosen to operate as the only technology throughout. When applying piecewise curves, Figure 11 shows that AHP is better suited for part load requirements, leaving the EC for almost exclusive use at its full load. Generally, there is more use of technologies in full/zero load configurations when piecewise curves are included. This means that a greater variety of technologies are purchased to avoid running any one of them at part load.

Purchased technology capacities also vary (figure 12). In both seasons, EC capacity is reduced in the piecewise results and AHP is purchased to account for the deficit. In the winter week, boiler size is also reduced, balanced by a larger heat storage capacity (table 5). Storage is used more in piecewise models, leading to lower cumulative system capacity. These results also show that the utility of the local distribution network is dictated by technology choices. For example, more power is distributed to the commercial properties in summer due to the purchase of a smaller mCHP. Heat networks are avoided. A small plant CHP is purchased in all cases, but it dumps heat in favour of distributing it. The system is limited in how much heat it can dump, so the plant CHP could be feasibly larger if that constraint were lifted.

Table 5. Capacity of distribution network to, and storage at, both demand locations. P = piecewise.

	Distribution						*Storage*					
	Gas		Heat		Power		Cooling		Power		Heat	
	SVE	*P*	*SVE*	*P*	*SVE*	*P*	*SVE*	*P*	*SVE*	*P*	*SVE*	*P*
Winter												
commercial	**1304**	1224	0	0	71	**75**	0	**24**	**7**	0	0	**59**
domestic	**69**	67	0	0	**41**	37	0	0	7	7	145	145
Summer												
commercial	**788**	622	0	0	40	**135**	0	**7**	7	7	230	**289**
domestic	6	**9**	**8**	0	40	**43**	0	0	7	7	5	**8**

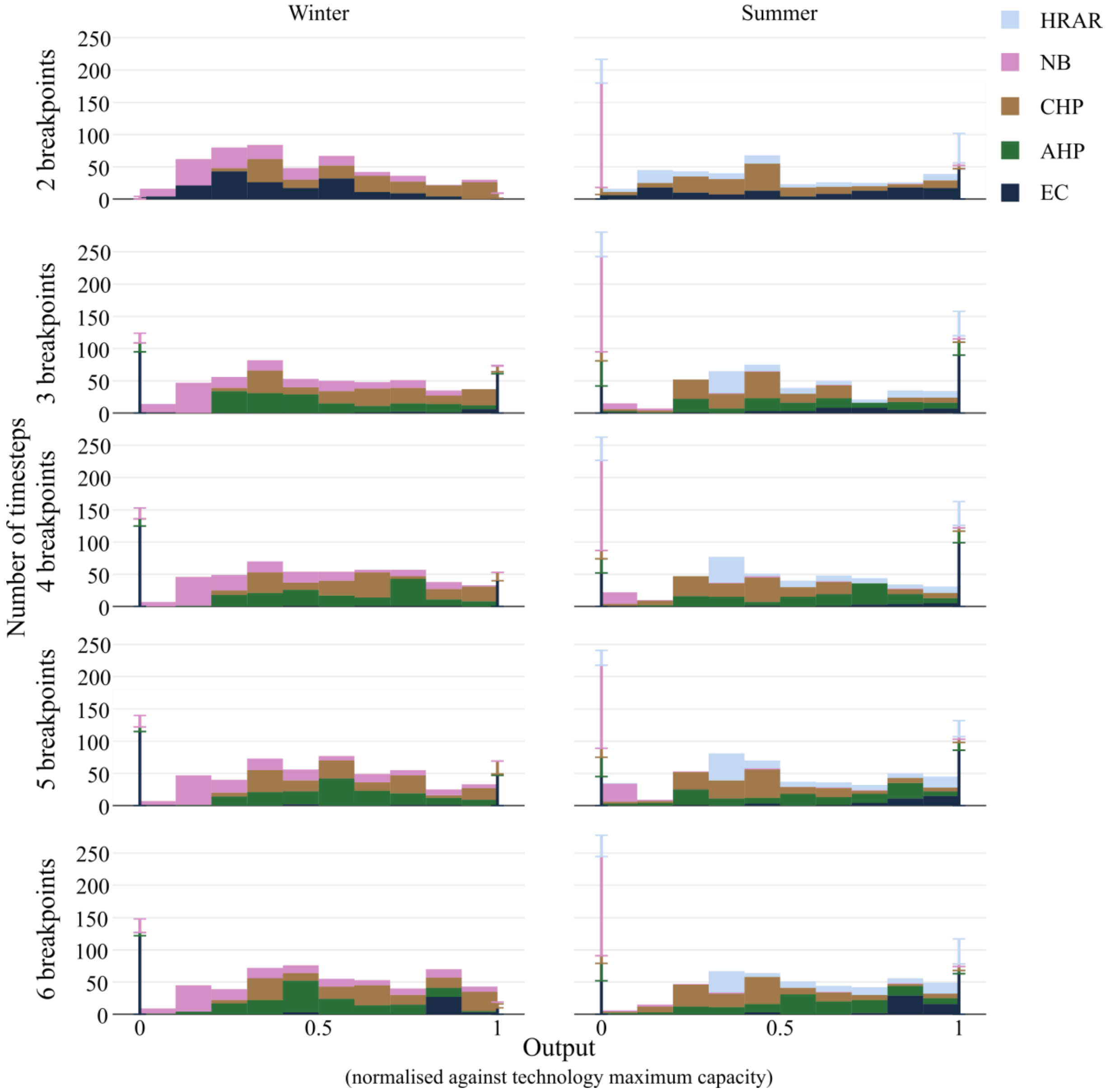


Figure 11. Technology output histograms, for SVE and optimised piecewise model runs. Full and zero loads are given as single points, with all other part load operation given in 10% increments.

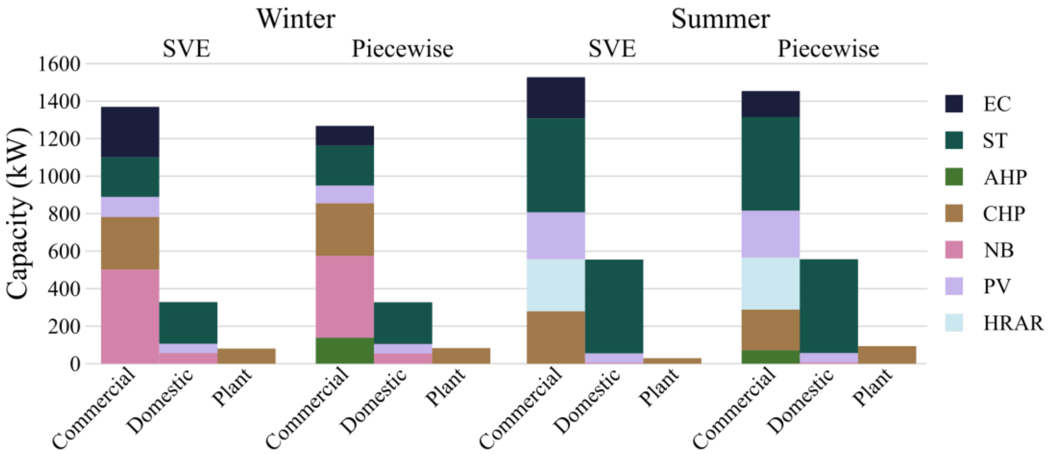


Figure 12[2]*. Energy supply technology investment portfolios at each location and in each season.*

[2] Only 3-breakpoint piecewise given, for clarity. Investment portfolios of piecewise models are all very similar, so figure 12 and table 5 can be taken as correct for all numbers of breakpoints.

5. Discussion

When choosing how to represent technology part-load characteristics, piecewise linearisation has an influence on the objective function value. By better matching the nonlinear curves, the optimal system is more expensive. This greater expense is more representative of the “real” cost of the optimal system and actually describes a cheaper system once the nonlinear curves have been applied ex-post. But, the improvement in accuracy comes with a solution time penalty. As increasing number of breakpoints does not greatly improve model accuracy, it is possible to limit the time penalty by going no further than three breakpoints. Following this, in winter the 2.5x time penalty is probably justifiable. In summer, the 15x time penalty may become unacceptable. Equally, choosing three equidistant piecewise breakpoints proves beneficial in the winter case (1.5x quicker than optimised) but certainly not in the summer case (4x slower than optimised). These three-breakpoint piecewise models have the same number of decision variables, which leads to agreement with Bischi et al. [3], that breakpoint positioning is likely an important factor. The central equidistant breakpoint is at 50% load rate, which requires more branches to be searched because there could be an optimal schedule with technologies operating either side. Conversely, the optimised central breakpoint, which tends towards 20-30% load rate, exists in a less critical part of the curve because so few possible solutions involve technologies operating at such low load rate. This problem of hopping either side of breakpoints is only exacerbated by a greater number of them, hence the solution time increasing with number of breakpoints.

On designing a piecewise model, assigning breakpoint locations to facilitate rapid convergence is difficult. Certainly, avoiding SOS2 and assigning breakpoints for strictly increasing/decreasing gradient is a good first step in reducing solution time, as number of decision variables is reduced. Further study is required to better understand the critical nature of breakpoints. One possible method is to run the model with a lower tolerance (e.g. 10% MIP gap). With this solution, the change in operation schedule will already be evident. Critical operating regions could then be identified, and breakpoints re-adjusted to avoid those regions. The design decision also depends on the purpose of the model: feasibility level studies could model with SVE to get a lower system cost, then apply nonlinear curves ex-post to get an upper bound. The piecewise system cost will exist within that range. Only on undertaking detailed design, for investment portfolio and operating schedule, would piecewise curves be necessary.

5.1. Three-dimensional piecewise linearisation

In this study, we linearised the 3D surface describing load rate, capacity, and consumption. Ideally, such a surface should be avoided as it increases problem complexity. However, there are advantages to using a continuous maximum capacity decision variable. Although real technologies exist in discrete sizes, it can be difficult to decide which ones to include for consideration in a model. If too many are included, the problem will become more complex than the continuous case, but too few might mean missing more optimal solutions. The continuous case removes the need to decide, giving a maximum capacity for which a designer can aim to find the closest available model. The relative benefits have not been tested in this study, but it is clearly a next step for investigation.

5.2. Justifying simplifications

To undertake several model runs, concessions were made. These should be retrospectively analysed where possible. First, the district network was simplified to just three locations, down from 21 (including intermediate transmission nodes), reducing decision variables by a factor of 20. Winter SVE and three-breakpoint optimised piecewise were tested with the full network. The objective function values were similar: £3880 and £4093 respectively, compared to £3989 and £4036 in the simplified network. The same trend occurred the investment decisions, whereby AHP is only purchased once piecewise is added. The difference is only evidenced by which technologies go where, particularly between the hotel and office. However, this benefit is accompanied by a two order of magnitude increase in solution time, to 209306 seconds. Second, piecewise linearisation was described by bounding constraints, rather than the more conventional SOS2. In this case, CPLEX was unable to continue the optimisation after more than 250000 seconds as hardware memory limits were reached. Convergence below 5% caused greatest computational difficulty.

6. Conclusion

This study analysed the use and optimisation of piecewise curves in an energy planning and operation problem. System cost converges on the "real" cost by representing nonlinear technology load curves as multiple straight lines, instead of assuming a single value efficiency. By comparing objective function value with system cost following ex-post application of nonlinear curves to the optimal operating schedules, piecewise curves reduce the difference from 12% to 0.69% on average. This reduction incurs a time penalty, from 1.6 times to 58 times longer to find an optimal solution when piecewise linearisation is used. Breakpoint positioning is a key factor in increased solution time, following solution branches either side of a breakpoint seems to create difficulty in model convergence. Increasing the number of breakpoints exacerbates this problem, leading to greater model solution time. The effect of optimising the placement of piecewise breakpoints, to reduce error relative to the nonlinear curve, does not necessarily improve the issue of convergence. In fact, in winter, simply placing three breakpoints equidistantly produces a solution quicker than its optimised counterpart. However, automation of breakpoint allocation allows for the creation of piecewise curves of strictly increasing/decreasing gradient. By doing so, solution time is reduced because special ordered sets can be avoided.

Understanding nonlinear consumption curves is insightful, whether or not they are incorporated into MILP optimisation by piecewise linearisation. They can be used at the feasibility level to get a system upper bound cost, by application ex-post to the operation schedule of an SVE model (the lower bound). Within such a range would lie the system cost for piecewise linearisation. This range could be utilised for system feasibility, but the effect of piecewise linearisation on technology capacities, use of storage and distribution networks, and operation schedules, is sufficiently distinct that detailed design would benefit from its inclusion. As such, further research is required to fully understand whether breakpoint allocation can be automated beyond simply minimizing fit error, to avoid convergence issues and ensure models solve in a practical time. This research will require analysis of solver parametrisation, such as the use of multiple runs with varying relaxation of the mixed integer optimality gap tolerance.

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Nomenclature

Technologies

AHP	Air source heat pump
B	Battery
CHP	Combined heat and power plant
EC	Electric chiller
GE	Grid electricity
HRAR	Heat recovery absorption refrigerator
mCHP	Micro CHP
NB	Natural gas boiler
PV	Solar photovoltaic panel
ST	Solar thermal panel
TES	Thermal energy storage

Optimisation:

MILP	Mixed integer linear programming
SLSQP	Sequential least squares programming
SOS2	Special ordered set of type 2

Other

SVE	Single value efficiency
HTP	Heat to power ratio

References

[1] M. Jennings, D. Fisk, and N. Shah, "Modelling and Optimization of Retrofitting Residential Energy Systems at the Urban Scale," *Energy*, vol. 64, pp. 220–233, Jan. 2014.

[2] B. Pickering, S. Ikeda, R. Choudhary, and R. Ooka, "Comparison of Metaheuristic and Linear Programming Models for the Purpose of Optimising Building Energy Supply Operation Schedule," in *12th REHVA World Congress*, 2016, vol. 6.

[3] A. Bischi, L. Taccari, E. Martelli, E. Amaldi, G. Manzolini, P. Silva, S. Campanari, and E. Macchi, "A Detailed MILP Optimization Model for Combined Cooling, Heat and Power System Operation Planning," *Energy*, vol. 74, pp. 12–26, 2014.

[4] Office of Energy Efficiency & Renewable Energy (EERE), "Commercial and Residential Hourly Load Profiles for All TMY3 Locations in the United States," United States Department of Energy.

[5] The Society of Heating Air-Conditioning and Sanitary Engineers of Japan (SHASE), *Computer Aided Simulation for Cogeneration Assessment & Design III*. Tokyo: Maruzen publishing, 2003.

[6] AECOM, Ed., *Spon's Mechanical and Electrical Services Price Book 2016*, 47 Har/Psc edition. CRC Press, 2015.

[7] J. Keirstead, N. Samsatli, and N. Shah, "SynCity: An Integrated Tool Kit for Urban Energy Systems Modelling," 2010, pp. 21–42.

[8] A. Omu, R. Choudhary, and A. Boies, "Distributed Energy Resource System Optimisation Using Mixed Integer Linear Programming," *Energy Policy*, vol. 61, pp. 249–266, Oct. 2013.

[9] L. Li, H. Mu, N. Li, and M. Li, "Economic and Environmental Optimization for Distributed Energy Resource Systems Coupled with District Energy Networks," *Energy*, vol. 109, pp. 947–960, Aug. 2016.

[10] S. Ikeda and R. Ooka, "Metaheuristic Optimization Methods for a Comprehensive Operating Schedule of Battery, Thermal Energy Storage, and Heat Source in a Building Energy System," *Applied Energy*, vol. 151, pp. 192–205, 2015.

[11] E. M. L. Beale and J. A. Tomlin, "Special Facilities in a General Mathematical Programming System for Non-Convex Problems Using Ordered Sets of Variables," *OR*, vol. 69, no. 447–454, p. 99, 1970.

[12] C. D'Ambrosio, A. Lodi, and S. Martello, "Piecewise Linear Approximation of Functions of Two Variables in MILP Models," *Operations Research Letters*, vol. 38, no. 1, pp. 39–46, Jan. 2010.

[13] A. Horner and J. Beauchamp, "Piecewise-Linear Approximation of Additive Synthesis Envelopes: A Comparison of Various Methods," *Computer Music Journal*, vol. 20, no. 2, pp. 72–95, 1996.

[14] P. Siriruk, "Fitting Piecewise Linear Functions Using Particle Swarm Optimization," *Suranaree J. Sci. Technol*, vol. 19, no. 4, pp. 259–264, 2012.

[15] D. Kraft and others, "A Software Package for Sequential Quadratic Programming," DFVLR Obersfaffeuhofen, Germany, DFVLR-FB–88-28, 1988.

[16] H. Wang, W. Yin, E. Abdollahi, R. Lahdelma, and W. Jiao, "Modelling and Optimization of CHP Based District Heating System with Renewable Energy Production and Energy Storage," *Applied Energy*, vol. 159, pp. 401–421, Dec. 2015.

[17] S. Ikeda and R. Ooka, "Optimal Operation of Energy Systems Including Energy Storage Equipment under Different Connections and Electricity Prices," *Sustainable Cities and Society*, vol. 21, pp. 1–11, Feb. 2016.

[18] A. Campos Celador, M. Odriozola, and J. M. Sala, "Implications of the Modelling of Stratified Hot Water Storage Tanks in the Simulation of CHP Plants," *Energy Conversion and Management*, vol. 52, no. 8–9, pp. 3018–3026, Aug. 2011.

[19] S. Pfenninger and J. Keirstead, "Renewables, Nuclear, or Fossil Fuels? Scenarios for Great Britain's Power System Considering Costs, Emissions and Energy Security," *Applied Energy*, vol. 152, pp. 83–93, Aug. 2015.

[20] IBM Corp., "IBM ILOG CPLEX Optimisation Studio." Armonk, NY, US, 2016.