Mitigating risk in district-level energy investment decisions by scenario optimisation

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Abstract

Increased availability of high resolution metered consumption data shows clear spatio-temporal variability in energy demand, both in terms of magnitude and time. This variability is rarely captured in district energy modelling and optimisation. In this paper, we demonstrate a modelling approach that integrates the stochastic variability of energy demand in energy system optimisation. In our set-up, energy demand is a stochastic function over time, separated into weekdays and weekends in a year. We consider cooling and electricity as end-uses. We implement the district energy optimisation using the mixed integer linear programming (MILP) Scenario optimisation (SO) framework. The stochastic variability of hourly demand is represented by 500 scenarios for 24 typical days in the year. For computational efficiency, we implement a scenario reduction step, resulting in 16 reduced scenarios as representative of the full scenario set. These 16 scenarios are used to formulate an SO model for a group of office buildings in Bangalore, India. The objective in this model is to minimise the Conditional Value at Risk (CVaR) associated with each scenario, weighted by the probability of that scenario being realised. A scenario can have some demand unmet, but this will incur a financial penalty. To better understand the necessary parametrisation of the model, the penalty for unmet demand is tested by sensitivity analysis.

Introduction

District energy systems are becoming more prevalent as a method of meeting building energy demand. Past work has shown that a district energy network gives better overall utility from energy systems than building-level solutions (Morvaj, Evins, and Carmeliet 2016; Li et al. 2016; Jennings, Fisk, and Shah 2014). However, optimal design of district energy systems requires good knowledge of the spatio-temporal variations of energy demand of buildings by their end-use. This information is not normally available, especially for unrealised buildings. Where available, past data from similar building typologies may be used. More commonly, archetype or reference demand profiles are assumed to be representative of future energy demand (Fonseca and Schlueter 2015; Swan and Ugursal 2009). Given that models for district energy optimisation can be computationally intensive, neither the stochastic variations nor future changes in demand are fully considered. Yet, it is well acknowledged that influences such as weather and occupancy can inevitably result in large variations in demand. Such variations are the cause of differences between projected and actual annual energy demand being recorded to be anywhere between 16% and 500% in UK commercial properties (The Carbon Trust 2011).

Methods for optimizing district energy systems that quantify resilience of the system against uncertainty in energy demand are still nascent. Indeed, the optimisation of multi-energy systems can be complex, with thousands of design variables. Scenario optimisation (SO), where risk measures are part of the objective function of optimisation, offers a mechanism of explicitly considering the uncertainty of system performance under variable demand scenarios. In SO models, a range of future scenarios is defined, each with a different probability of occurrence. The objective function aims to best meet all possible future realisations of the system, with reasonable consideration for a scenario's probability of occurrence. SO is hitherto applied in the context of large-scale energy systems modelling. For instance, Maurovich-Horvat, Rocha, and Siddiqui 2016 used it for the optimal operation of a combined heat and power plant under uncertain spot pricing, while Bukhsh, Papakonstantinou, and Pinson 2016 applied it to energy market clearing under uncertain wind availability.

This paper extends the SO methodology to uncertain energy demand when designing district energy systems. To do so, a set of probabilistic demand scenarios must be generated as input data. There is an infinite number of future scenarios that could be realised, each with an infinitesimally small probability of occurrence and a probability density function with which they can collectively be described. We cannot consider an infinite number of scenarios, so reduction techniques are instead employed. By randomly sampling a stochastic representation of demand, a large number of equiprobable scenarios can be created. In order to maintain tractability of the optimisation process, these scenarios can be clustered, allowing for further discretisation without loss of understanding. In the following sections we describe a method for using data-driven stochastic energy demand profiles to produce scenarios for use in SO. As a preliminary test of the applicability of SO to uncertain demand, we apply the reduced scenarios to a fictional collection of office buildings in India. A comparison is made between single-scenario and multi-scenario cases. We also analyse the sensitivity of the outcomes to the pre-imposed penalty for unmet demand scenarios.

Case study

District

Stochastic energy demand profiles have been derived from high-resolution metered data of a desk-based office building in Bangalore, India (the methodology for generating these profiles is described in Ward et al. 2016). A collection of office buildings within the same city are assumed to define a district. Figure 1 shows each of these buildings, and the nodes used to represent them. In most cases several buildings have been merged into one node on the district cooling network. Building floor area (table 1) has been inferred from the external footprint and number of floors for each building. *No other information is known about these buildings*; we use only their relative size and position to test our modelling approach. Building occupancy is one of the uncertainties which leads to the variability of the metered data. We assume a similar degree of uncertainty in the buildings of our test district, hence we do not attempt to infer occupancy. Four energy centres are proposed, at different positions on the periphery of the district. Each is sited on a currently undeveloped piece of land, according to satellite data. The cooling network is given in figure 1, with an assumption of no losses in the pipework, due to the similarities between ground temperature and cooling water temperature.

Demand profiles

Building demand for cooling and electricity has been randomly sampled from the stochastic energy demand profiles and scaled to building floor area. 500 samples have been taken for 24 'typical' days. These days represent weekends and weekdays in each month of the year. Although the use of typical days is prominent in energy optimisation (e.g. Omu et al. 2015; Jennings, Fisk, and Shah 2014; Cano et al. 2014) their use leads inevitably to inaccuracies compared to the full time series (Pfenninger 2017). However, we explicitly account for variability within typical days and therefore do not expect significant difference between using typical days versus full time series of demand. Figure 2 shows the possible variation in demand within each of the 24 typical days, summed over the entire district. Variation at a building level (not shown here for brevity) is more prominent because

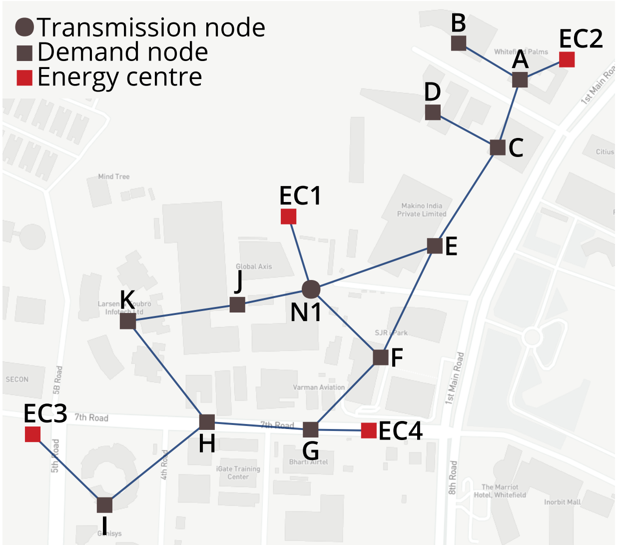

Figure 1: Case study district

the aggregation of demand over the entire district results in a certain degree of load smoothing.

Available technologies

Table 1 describes technologies which are available at each node. Although buildings have the option to have their own building-level air conditioning (AC), this is limited to less than the total demand for the building. It is expected that cooling demand in this district will be met via the cooling network, by either a large-scale electric chiller (EC) or an absorption chiller, linked to a combined heat and power plant (CCHP). Cold water thermal energy storage (TES) is available at energy centres, to provide flexibility, and building-level photovoltaic solar panels (PV) can be installed, which are inherently inflexible. Technology costs are calculated from various sources. Where costs specific to India were not available, values from the SPONS mechanical and electrical services price book (AECOM 2015) have been converted from GBP to INR at a rate of 90 INR/GBP. A full specification of technology costs can be found in the model configuration at `https://github.com/brynpickering/bangalore-calliope`.

Weather

Solar photovoltaic panels have the capability to meet a large proportion of the electrical demand in the studied district. Expected output is based on solar irradiance and temperature for 2015 and is acquired in steps from `www.renewables.ninja` (Pfenninger and Staffell 2016). As the metered data is from December 2015 to November 2016, the weather variability experienced across that year, including extreme weather events, will be encompassed in the range of metered demand profiles. Thus, weather variability will be a component of the scenarios we produce from stochastic samples.

Table 1: District node details.

Node	A	B	C	D	E	F	G	H	I	J	K	EC1	EC2	EC3	EC4	N1
GIA	5,440	36,586	12,650	22,400	17,184	78,086	46,582	93,064	23,846	178,496	39,504	N/A				N/A
Roof area	2,720	6,098	3,162	5,600	8,592	11,155	11,646	18,613	5,962	22,312	7,900					
Technologies	PV, AC, Grid											EC, CCHP, TES, Grid				

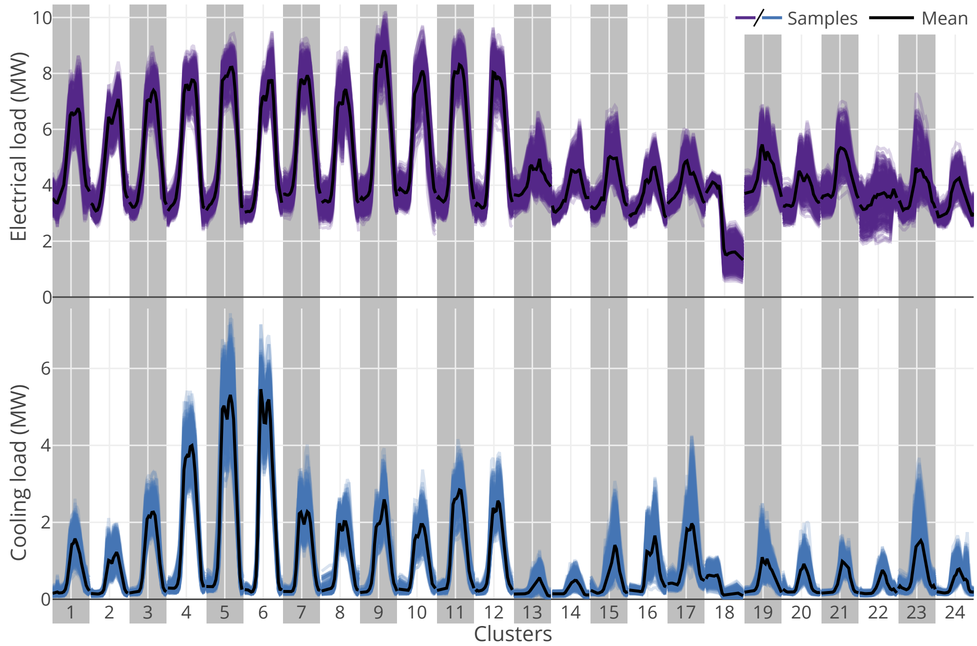

Figure 2: Demand samples and cluster means, summed over all locations in the district. Clusters represent weekday (clusters 1-12) and weekends (clusters 13-24) in each month, starting in December.

Methodology

Scenario reduction

The introduction of probabilistic scenarios in an optimisation model can result in model intractability. Particularly if there are hundreds of scenarios, it is not feasible to consider anything other than basic scenario optimisation models. Accordingly, the selection of the 'right' subset of scenarios becomes an important step in SO models. Conejo, Carrión, and Morales 2010 proposed the use of probability distances to reduce the number of model scenarios, detailing two primary variants of the method which use the *Kantorovich distance*[1]. Both variants apply a cost metric to each scenario s in the scenario set S, from which a subset S' is chosen based on the minimisation of the difference in the probability distributions describing the costs in S and S'.

The two variants of scenario reduction proposed by Conejo, Carrión, and Morales 2010 differ on the cost metric applied to each scenario. In the **first**, a key performance indicator (KPI) describing the scenarios is selected. This might be the maximum hourly demand per scenario, or the total demand over the entire year. The problem here is the inability to assess which of the possible KPIs is actually *key* from the point of having the biggest impact on the objective function. The **second** variant requires that a simple (non probabilistic) optimisation model is run for each scenario. These simple models are a formulation of the SO model which do not consider uncertainty. They are relatively fast to solve, and can be run in parallel on a high performance cluster in a matter of minutes. As a result of the simple optimisation, an objective function is calculated for each scenario. It is used in the Kantorovich distance calculation.

A refinement of the second variant, proposed by Bruninx and Delarue 2016, is used in this study. Conejo, Carrión, and Morales only considered optimisation of operation costs in its second variant, fixing the investment cost for each of the simple models. Bruninx and Delarue included investment costs in addition to operation costs. Therefore all decision variables are part of the simple model optimisation, but binary and integer constraints are not included for computational efficiency. In our simple models, We keep the binary 'purchase' constraints applied to investment decisions. Because we use typical days as against full time series of annual demand, our simple optimisation runs within reasonable solution times ($O(100s)$ on a high performance cluster).

The process for scenario reduction can be summarised as follows:

1. Create 500 scenario models, in which each building is randomly assigned electricity and cooling

[1] For a detailed mathematical formulation, readers are referred to Conejo, Carrión, and Morales 2010; Römisch 2009.

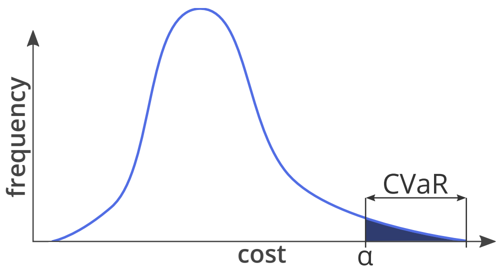

Figure 3: Representation of the CVaR risk measure.

demand profiles for each typical day.
2. Run simple optimisation for each scenario in parallel, minimising system cost (investment and operation) for each case independently.
3. Select 16 scenarios to represent the 500 input scenarios, by minimising the Kantorovich distance between the probability density of their objective function values to that of the full scenario set.
4. Assign each scenario in the full set to the closest (by probability distance) of the 16 scenarios in the reduced subset, weighting each reduced scenario by the number of scenarios it represents.

Scenario optimisation

Once reduced scenarios are derived, the uncertainty described by these scenarios can be dealt with by scenario optimisation (SO). Our SO model has two stages. The first involves finding the optimal technologies and their capacities, irrespective of their ability to meet variability in demand. In the second stage, the optimal technologies are reassessed for their ability to meet variability in energy demand represented by the 16 reduced scenarios. If energy demand represented by any particular scenario is not met, a financial penalty is incurred. The impact of a single scenario is weighted by its probability of occurrence, such that low probability scenarios may have unmet demand without incurring a large penalty on the overall objective function.

Having unmet demand is a risk, which we monetise for direct application to the objective function. The risk metric we use is the Conditional Value at Risk (CVaR). The monetary cost of unmet demand scenarios in the SO model can be described by a probability distribution. Given a confidence level $\alpha \in [0, 1)$, the CVaR describes the sum of the expected cost above that level (fig. 3). As it concentrates on the right-hand tail of the distribution, it is a risk measure that is heavily influenced by the worst-case scenarios. It is an extension of the value at Risk (VaR) measure, which minimizes the cost at the confidence level α, that is better suited to linear optimisation models (Rockafellar and Uryasev 2002).

Model formulation

The SO objective function is shown below in eq. 1. It includes both initial investment as well as operational cost of technologies and is a variant of the objective function used by Maurovich-Horvat, Rocha, and Siddiqui 2016. In this formulation, CVaR of a scenario is estimated using η, weighted by the probability of occurrence of that scenario, $P_{s'}$, and calculated across scenarios by combination with the model-wide VaR, ξ. Model-wide parameters α and β describe the confidence interval and risk aversion metric, respectively. In this study, α was set to 90% and β to 5.

$$\min \sum_{s' \in S'} (P_{s'} \boldsymbol{cost}_{s'}) + \beta(\boldsymbol{\xi} + \frac{1}{1-\alpha} \sum_{s \in S'} (P_{s'} \boldsymbol{\eta}_{s'})) \quad (1)$$

The objective function is subject to various constraints typical to energy system models, all of which are formulated within the open-source MILP modelling framework Calliope v0.6.0 (release candidate) (Pfenninger and Keirstead 2015)[2]. Models were run on a high performance computing cluster, with optimisation undertaken by the Gurobi solver (v6.0.2).

Results

Scenario reduction

The 500 scenarios run through the simple optimisation model have total annual system energy demand ranging from 50GWh to 53.8GWh and the objective function values from 439×10^6 to 519×10^6 INR. Although there is a trend towards greater objective function value with increased total system demand (fig. 4), the highest objective value is elicited by a system demand of 52.35GWh and the lowest by a system demand of 51.25GWh.

Figure 5 shows the reduced scenarios: the total annual system demand for these ranges from 50.6 GWh to 52.75 GWh. Note that the scenario representing the highest objective function values (218) has a total system demand lower than ten other reduced scenarios. Table 2 gives further details on the reduced scenarios. The weight assigned to each is the percentage of the 500 initial scenarios that they represent and will be used as the scenario probability in the SO model. This weight ranges from 1.6% to 9.8%. The undulation of weight matches the non-normal shape of the objective function value distribution (shown in fig. 4).

Scenario optimisation

As a baseline, the mean energy demand (seen in fig. 2) was used with the simple (non probabilistic) optimisation model. Fig. 6 shows the installed capacity of technologies for this baseline. Centralised electric and absorption chillers meet most of the of the system cooling demand, with some building-level AC installed alongside. Electricity demand is mainly met by the grid, although approximately a third of it is met by solar photovoltaic panels installed on building roofs. We are not connecting the nodes by power

[2] The entire model, including input parameters, stochastic curves and optimisation constraints, can be found at `https://github.com/brynpickering/bangalore-calliope`.

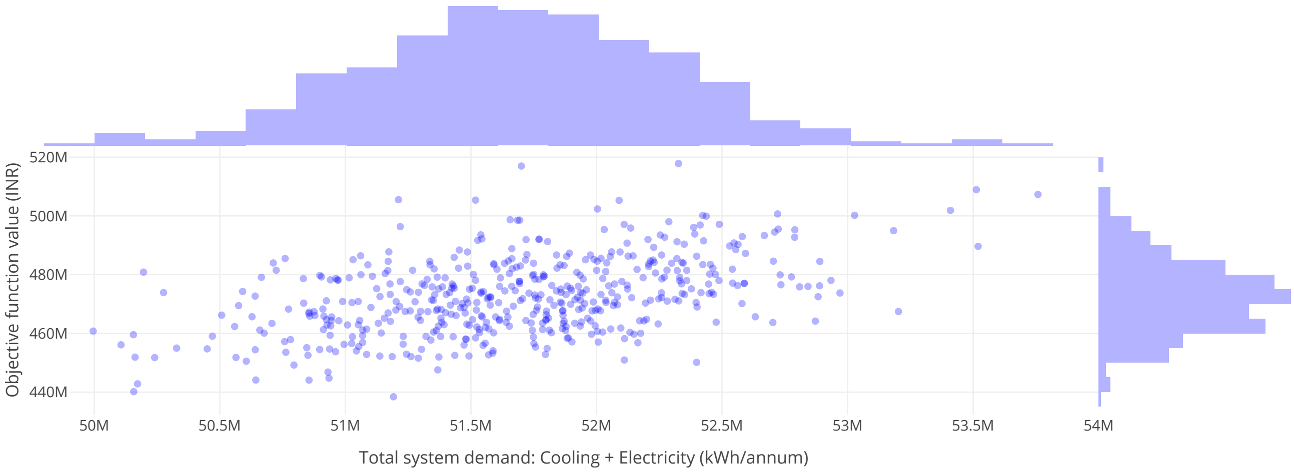

Figure 4: Total district demand against deterministic objective function value for 500 sets of sampled demand profiles. Distribution of demand and objective function value given outside the scatter plot.

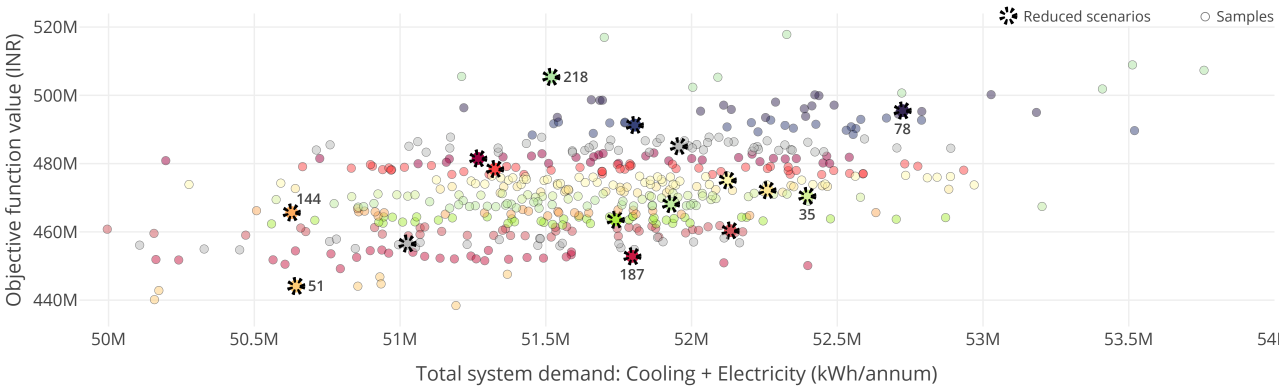

Figure 5: Total district demand against deterministic objective function value for 500 sets of sampled demand profiles, coloured by reduced scenario group. All scenarios within a reduced scenario group will be represented by a single reduced scenario in subsequent scenario optimisation. Reduced scenarios representing each group, 16 in total, are emphasised, and occasionally labelled.

Table 2: Details of 16 reduced scenarios which represent the full 500 scenario set.

Scenario	**Cost (10^8 INR)**		**Weight**
	Total	Investment	
51	4.44	1.56	1.6%
187	4.53	1.66	5.6%
221	4.57	1.58	5.0%
320	4.60	1.57	7.8%
4	4.64	1.57	8.4%
144	4.66	1.76	5.2%
391	4.68	1.58	7.0%
35	4.70	1.71	7.0%
275	4.72	1.75	6.2%
206	4.75	1.56	9.8%
430	4.78	1.77	9.6%
423	4.81	1.67	6.2%
147	4.85	1.83	9.8%
491	4.91	1.85	4.6%
78	4.96	1.86	4.2%
218	5.05	2.02	2.0%

lines. Instead, the electricity produced by the CCHP at the energy centres is exported back to the grid. The result of our SO model shows an increase in energy centre capacity in EC1 (fig. 7). The TES capacity does not increase dramatically, although it is an ideal technology to increase system flexibility. PV investment decreases considerably when accounting for uncertainty. This is because the PV modules are being sized to the lowest peak electricity demand of all scenarios (we do not allow electricity from PVs to be exported back to the grid). As a result, there is greater reliance on the electricity grid.

Unmet demand penalty

The financial penalty for unmet demand is varied from 100 INR/kWh to 10^7 INR/kWh, in increments of a factor of ten. The result, as expected, is decrease in unmet demand with increased penalty. Fig. 8 shows, for each scenario, the magnitude of decrease in unmet demand across all 16 scenarios for the penalty range between 100 INR/kWh and 10^6 INR/kWh. At 10^7 INR/kWh there is no unmet demand. Unmet demand is found uniformly across scenarios at penalty

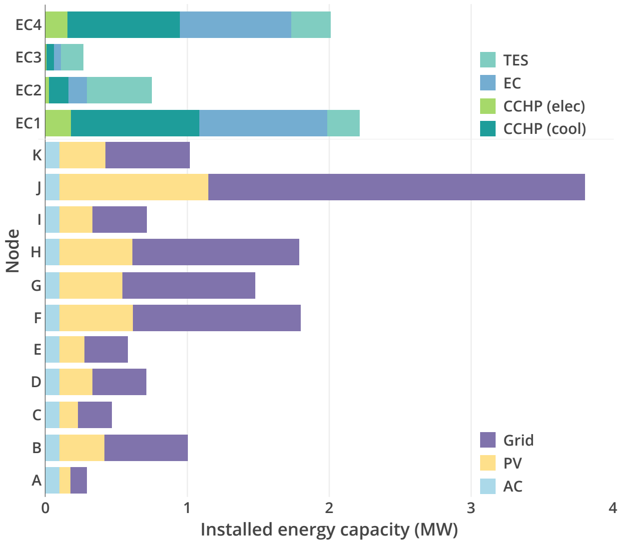

Figure 6: Optimal configuration of installed capacity in the mean (single-scenario) solution.

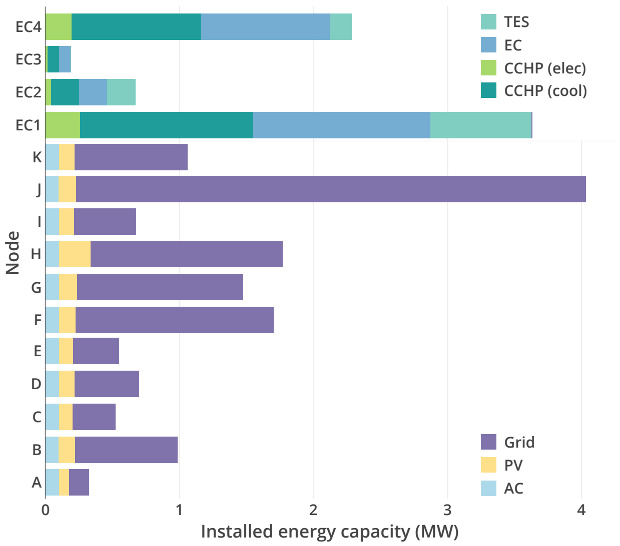

Figure 7: Optimal configuration of installed capacity resulting from an SO model with unmet demand penalty of 10^5 INR/kWh.

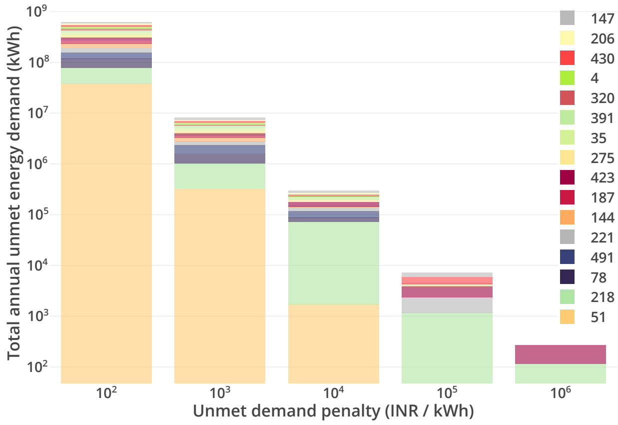

*Figure 8: Comparison of quantity of unmet demand incurred in various scenarios for penalty rates ranging from 100 to 10^6 INR/kWh. **NOTE**: Scenarios are given in ascending order of probability, but as the y axis is a $\log_{10}$ scale, lower probability scenarios appear to have a greater contribution than is actually the case.*

Table 3: Comparison of system investment costs (10^8 INR) between the non-SO case (mean) and the SO case with varying penalty for unmet demand.

	SO, with penalty (INR / kWh):					
Mean	**10^2**	**10^3**	**10^4**	**10^5**	**10^6**	**10^7**
1.54	0.11	0.88	1.51	1.86	1.89	1.89

levels 10^2 and 10^3 INR/kWh. Only at 10^4 INR/kWh do we see a particular weighting of unmet demand onto lower probability scenarios. Large amounts of unmet demand allow for the technology capacities to be reduced. Table 3 shows an increasing investment cost with increasing unmet demand penalty. However, the investment cost is similar from a penalty of 10^5 onward. At the higher penalty values, the optimal system investment cost is 1.89×10^8 INR.

Discussion

It is clear that the financial penalty used in SO has a significant effect. If it is too low then it has no influence on the objective function. Setting it too high would be similar to solving the optimisation project for worst case scenarios (i.e. robust optimisation). In other SO studies, the penalty has been an interpretable value/cost. For instance, Bukhsh, Papakonstantinou, and Pinson 2016 used the costs of dispatchable, expensive diesel generators in cases of insufficient wind supply. Maurovich-Horvat, Rocha, and Siddiqui 2016 use the higher cost spot electricity and gas prices as a penalty for incorrectly purchased energy from the futures market. Both these penalties exist in their respective markets. In our case, the penalty represents a virtual source of energy supply, to ensure all demand is met. This might translate in reality to a diesel generator for electricity, or window-mounted AC for cooling. Neither is likely to incur the costs per kWh that we specify as penalty to avoid their use ($\geq 10^5$ INR / kWh), especially if we consider that the cost of diesel and grid electricity is around 60 INR / kWh and 8 INR / kWh, respectively. This cost therefore must monetize and include factors such as commercial losses, loss in productivity, and cost of carbon. Losses in commercial sales due to grid-based power outages can be up to 2% in Bangalore (Dollar, Hallward-Driemeier, and Mengistae 2005). Indeed, if we consider CVaR as the 'cost of carbon', we may better penalise highly dispatchable but polluting technologies.

Finally, this study has considered a stable grid, but the reality is daily power outages which are met by diesel generators. In fact, in the SO cases, less PV is installed, in favour of greater grid capacity. Power outages would perhaps leave these systems *more* vulnerable than if we designed to the single-scenario solution.

Conclusions

In this study, we have sampled data-driven stochastic demand profiles for an office in Bangalore, India, and applied the resulting 500 scenarios to a fictional district of office buildings in the same city. By minimising probability distances we were able to consider a subset of 16 scenarios as representative of the full 500 scenario set. This subset of scenarios allowed us to create a tractable Mixed Integer Linear (MILP) scenario optimisation (SO) model, using Conditional Value at Risk (CVaR) as our risk measure. In so doing we were able to seek the optimal technology investment portfolio which would meet all possible future demand profiles, with due consideration of the probability of those futures being realised. The exogenous penalty for unmet demand, used to calculate CVaR, was varied between 10^2 and 10^7 INR / kWh to assess its impact on the optimal technology investment portfolio. This penalty represents the cost incurred for unmet demand with an external supply technology, or in the cost of discomfort / loss of productivity caused by leaving it unmet. It is clear that a greater understanding of the realistic penalty for unmet demand is required, as its impact varies considerably. At a low penalty, closer to the actual cost of bringing in building-level technological solutions, there is a considerable amount of unmet demand. At a very high penalty, there is no or little unmet demand, which means we are only optimising for the worst case.

Future work

Greenhouse gas penalty

By allowing all demand to be met, but a cost of greater emissions for greater flexibility (e.g. diesel generators), we would be able to use a cost of carbon as our SO risk measure. Testing the cost of carbon would then give us an indication of the required level to tip the balance towards low-carbon technologies in SO.

Power outages

Given the geographic context, it is prudent to also consider a non-infinite grid supply. Particularly, the effect of outages on previously designed optimal solutions, such as those from this study, and the effect on future SO models.

Scenario reduction

It is not clear how well scenario reduction has allowed our SO model to cover the scenario space. As such, it will be necessary to test the SO model results against the full scenario set. This is undertaken by fixing the technology capacities, as given by the SO optimal solution, and running each of the 500 scenarios in a rolling horizon optimisation. If there is too much unmet demand in any scenarios, this will indicate that the scenario reduction was unsuccessful. If that is the case, we will increase the number of reduced scenarios and test out skewing the chosen scenarios towards the higher values of the scenario set objective functions. In the latter case, this will disproportionately represent the higher values in more detail than the lower values. As we are only particularly interested in controlling the extreme cases, this may be a useful measure. We may find that 16 scenarios is more than sufficient, at which point we can test fewer reduced scenarios, to allow greater complexity in other dimensions.

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References

AECOM, ed. (2015). *Spon's Mechanical and Electrical Services Price Book 2016*. 47 Har/Psc edition. CRC Press. 840 pp. ISBN: 978-1-4987-3506-3.

Bruninx, K. and E. Delarue (2016). "Scenario Reduction Techniques and Solution Stability for Stochastic Unit Commitment Problems". In: *2016 IEEE International Energy Conference (ENERGYCON)*. 2016 IEEE International Energy Conference (ENERGYCON), pp. 1–7. DOI: 10.1109/ENERGYCON.2016.7514074.

Bukhsh, W. A., A. Papakonstantinou, and P. Pinson (2016). "A Robust Optimisation Approach Using CVaR for Unit Commitment in a Market with Probabilistic Offers". In: *2016 IEEE International Energy Conference (ENERGYCON)*. 2016 IEEE International Energy Conference (ENERGYCON), pp. 1–6. DOI: 10.1109/ENERGYCON.2016.7514076.

Cano, Emilio L. et al. (2014). "A Strategic Optimization Model for Energy Systems Planning". In: *Energy and Buildings* 81, pp. 416–423. ISSN: 0378-7788. DOI: 10.1016/j.enbuild.2014.06.030.

Conejo, Antonio J., Miguel Carrión, and Juan M. Morales (2010). *Decision Making under Uncertainty in Electricity Markets*. Vol. 1. Springer.

Dollar, David, Mary Hallward-Driemeier, and Taye Mengistae (2005). "Investment Climate and Firm Performance in Developing Economies". In: *Economic Development and Cultural Change* 54.1, pp. 1–31. ISSN: 0013-0079. DOI: 10.1086/431262.

Fonseca, Jimeno A. and Arno Schlueter (2015). "Integrated Model for Characterization of Spatiotemporal Building Energy Consumption Patterns in Neighborhoods and City Districts". In: *Applied Energy* 142, pp. 247–265. ISSN: 0306-2619. DOI: 10.1016/j.apenergy.2014.12.068.

Jennings, Mark, David Fisk, and Nilay Shah (2014). "Modelling and Optimization of Retrofitting Residential Energy Systems at the Urban Scale". In: *Energy* 64, pp. 220–233. ISSN: 03605442. DOI: 10.1016/j.energy.2013.10.076.

Li, Longxi et al. (2016). "Economic and Environmental Optimization for Distributed Energy Resource Systems Coupled with District Energy Networks". In: *Energy* 109, pp. 947–960. ISSN: 0360-5442. DOI: 10.1016/j.energy.2016.05.026.

Maurovich-Horvat, Lajos, Paula Rocha, and Afzal S. Siddiqui (2016). "Optimal Operation of Combined Heat and Power under Uncertainty and Risk Aversion". In: *Energy and Buildings* 110, pp. 415–425. ISSN: 0378-7788. DOI: 10.1016/j.enbuild.2015.11.009.

Morvaj, Boran, Ralph Evins, and Jan Carmeliet (2016). "Optimising Urban Energy Systems: Simultaneous System Sizing, Operation and District Heating Network Layout". In: *Energy* 116, Part 1, pp. 619–636. ISSN: 0360-5442. DOI: 10.1016/j.energy.2016.09.139.

Omu, Akomeno et al. (2015). "Economic, Climate Change, and Air Quality Analysis of Distributed Energy Resource Systems". In: *Procedia Computer Science* 51, pp. 2147–2156. ISSN: 18770509. DOI: 10.1016/j.procs.2015.05.487.

Pfenninger, Stefan (2017). "Dealing with Multiple Decades of Hourly Wind and PV Time Series in Energy Models: A Comparison of Methods to Reduce Time Resolution and the Planning Implications of Inter-Annual Variability". In: *Applied Energy* 197, pp. 1–13. ISSN: 0306-2619. DOI: 10.1016/j.apenergy.2017.03.051.

Pfenninger, Stefan and James Keirstead (2015). "Renewables, Nuclear, or Fossil Fuels? Scenarios for Great Britain's Power System Considering Costs, Emissions and Energy Security". In: *Applied Energy* 152, pp. 83–93. ISSN: 0306-2619. DOI: 10.1016/j.apenergy.2015.04.102.

Pfenninger, Stefan and Iain Staffell (2016). "Long-Term Patterns of European PV Output Using 30 Years of Validated Hourly Reanalysis and Satellite Data". In: *Energy* 114, pp. 1251–1265. ISSN: 0360-5442. DOI: 10.1016/j.energy.2016.08.060.

Rockafellar, R. Tyrrell and Stanislav Uryasev (2002). "Conditional Value-at-Risk for General Loss Distributions". In: *Journal of Banking & Finance* 26.7, pp. 1443–1471. ISSN: 0378-4266. DOI: 10.1016/S0378-4266(02)00271-6.

Römisch, Werner (2009). "Scenario Reduction Techniques in Stochastic Programming". In: *Stochastic Algorithms: Foundations and Applications*. Ed. by Osamu Watanabe and Thomas Zeugmann. Vol. 5792. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 1–14. ISBN: 978-3-642-04943-9 978-3-642-04944-6. DOI: 10.1007/978-3-642-04944-6_1.

Swan, Lukas G. and V. Ismet Ugursal (2009). "Modeling of End-Use Energy Consumption in the Residential Sector: A Review of Modeling Techniques". In: *Renewable and Sustainable Energy Reviews* 13.8, pp. 1819–1835. ISSN: 1364-0321. DOI: 10.1016/j.rser.2008.09.033.

The Carbon Trust (2011). *Closing the Gap: Lessons Learned on Realising the Potential of Low Carbon Building Design*. The Carbon Trust.

Ward, Rebecca et al. (2016). "Data Driven Bottom-up Approach for Modelling Internal Loads in Building Energy Simulation Using Functional Principal Components". In: *Proceedings of the 3rd IBPSA-England Conference BSO 2016*. BSO 2016. Great North Museum, Newcastle.